

~~Short cut at /a/GKX?~~

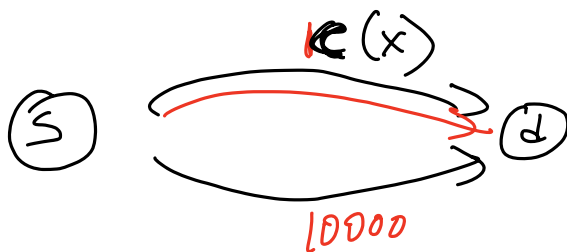
Last time: algorithms on graphs

- ① please decide on paper + submit GRM
try 10+ days before presentation
 - ② sign up ^{as} discussant
 - ③ Showing up is important.
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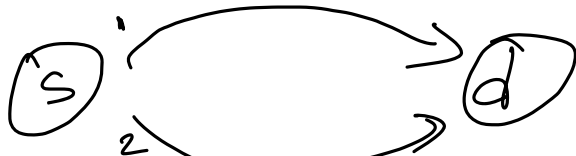
Today: games + incentives on networks

- ① what road do I take → ① congestion pricing
 - ② who incentivized to friend request to on authority
LinkedIn
-

routing decisions: what road do I take
when driving somewhere?



$$c_1(x) = 5x$$



X : fraction of people
taking an edge.

$$X_1 + X_2 = 1$$

$$C_2(x) = 2 + .001x$$

what is x_1 and x_2 in Nash equilibrium?

Nash

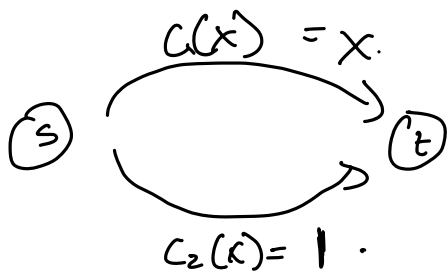
$$C_1(x_1) = C_2(x_2)$$

$$5x_1 = 2 + .001x_2$$

$$x_1 + x_2 = 1$$

$$x_1^*, x_2^*$$

Solve system of equations



$$x_2 = 1 - x_1$$

$$x_1^* = 1 \rightarrow \text{Nash equilibrium}$$

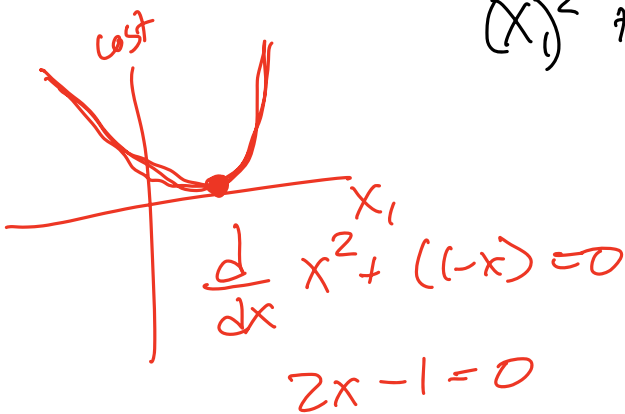
$$x_1 + x_2 = 1 \Rightarrow x_2^* = 0$$

everyone takes air train

average time off island

$$x_1(x_1) + x_2(1)$$

$$(x_1)^2 + x_2 = (x_1)^2 + (1 - x_1) = 0$$



derivative

$$2x_1^* - 1 = 0$$

$$x_1^* = 1/2$$

$$x_2^* = 1/2$$

social optimal

$$\left(\frac{1}{2}\right)^2 + \left(1 - \frac{1}{2}\right) = \frac{1}{4} + \frac{1}{2}$$

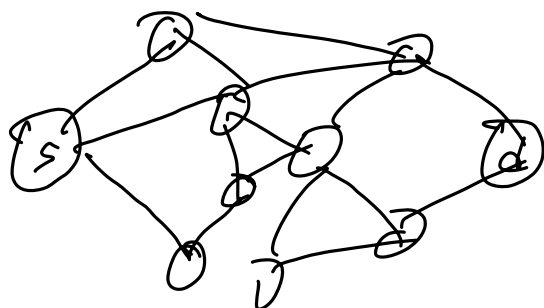
best possible social cost $\rightarrow = \frac{3}{4}$

Cost under Nash: $1^2 + 0 = 1$

Price of anarchy?

$$\frac{\text{Cost under Nash}}{\text{Cost under best}} = \frac{1}{\frac{3}{4}} = \frac{4}{3}$$

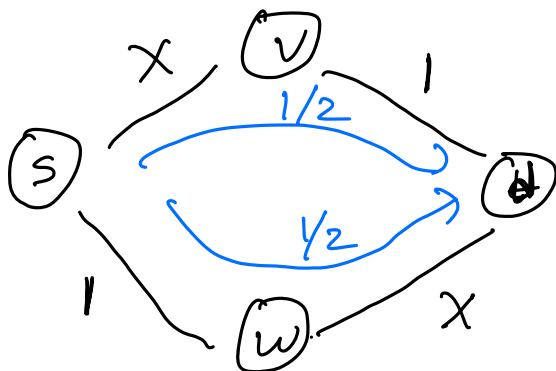
How bad can price of anarchy be?



$$\frac{4}{3}$$

if

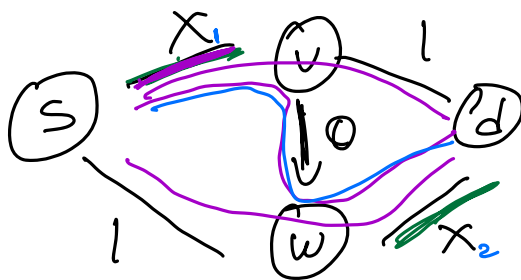
Cost functions are linear
 $C(x) = ax + b$



What is Nash equilibria?

$$\frac{1}{2} \left(1 + \frac{1}{2} \right) + \frac{1}{2} \left(1 + \frac{1}{2} \right) = \frac{3}{2}$$

What happens if I add a road?



$$\text{middle path} = x_1 + x_2$$

$$\begin{aligned} \text{bottom} &= 1 + x_2 \quad \leftarrow \\ \text{top} &= x_1 + 1 \quad \leftarrow \end{aligned}$$

everyone takes middle:
in equilibrium $x_1 = 1$
 $x_2 = 1$

everyone pays 2

price of anarchy = $\frac{4}{3}$ $\frac{2}{3/2} = \frac{4}{3}$

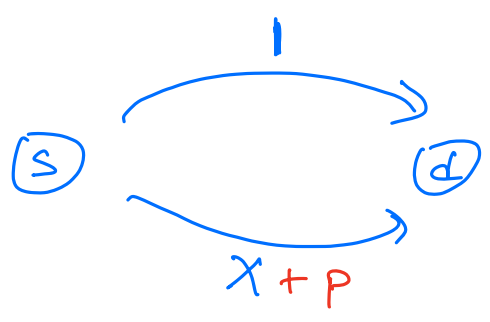
Braess paradox

Adding a road can increase traffic.
(even holding fixed # of drivers)

→ traffic planning - is your road actually going to decrease traffic?

• power grids - adding a transmission line makes things worse.

Congestion pricing



$$\begin{aligned} X(p) \\ 1 &= x+p \\ x &= 1-p \end{aligned}$$

what should p be?

• minimize average travel time?

• $x^* = 1/2$ $1/2$

$$\Rightarrow 1 = \cancel{X} + p \Rightarrow \boxed{p = 1/2}$$

• raise the most money?

$$\begin{aligned} \text{Revenue}(x) &= p \cdot \cancel{x(p)} \quad \# \text{ of ppl taking} \\ &= p(1-p) \quad \text{bottom road} \\ &\quad \text{when cost is } p. \\ \Rightarrow p^* &= 1/2 \end{aligned}$$