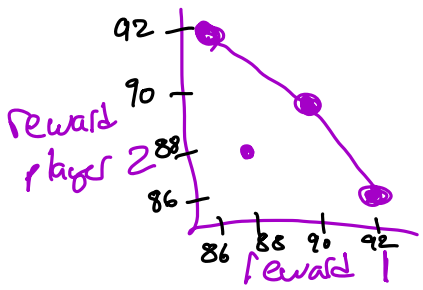
	P	E
Presentation	180 • 90, 90	178 • 86, 92
Exam	178 • 92, 86	176 • 88, 88



Nash Equilibria: (E, E)

Pareto optimal: (P, P), (E, P), (P, E)

Social optimal: (P, P)

Price of anarchy: $\frac{180 - 176}{180} = \frac{4}{180}\%$

pair of actions where no one wants to deviate.

There exists no other pair of actions where one person is better off and no one is worse off.

Sum of rewards is maximized

Intro to Auctions

- Suppose you want to sell a single item to a bunch of people = bidders
- Each bidder i is going to bid $\$ b_i$
- You need to decide:
  - who gets the item
  - how much do they pay?

1st price auction: - give to $\boxed{\text{argmax}_i\ b_i}$

- Charge them $\boxed{b_i}$

2nd price: - give to $\boxed{\text{argmax}_i\ b_i}$

- now charge $\boxed{\text{second item 2nd highest bid}}$

all pay auctions: - give it to highest bidder

- charge everyone their bid.

Rewards

Each bidder has a value for item v_i

Value for each dollar +1

$\Rightarrow$ if get item and pay p_i

utility $= v_i - p_i$

$\Rightarrow$ if I don't get item and don't pay

utility $= 0$

First price auctions

You are someone who values item at v_i

what should you bid?

what should b_i be?

$0 \le b_i \le v_i$ ✓

$$\text{utility} = \begin{cases} v_i - b_i & \text{if I win} \\ 0 & \text{if I lose} \end{cases}$$

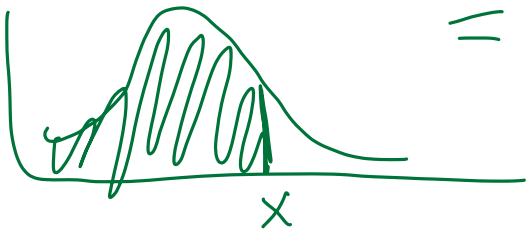
$f(x)$ = my expected utility if I bid "x"

$$= \begin{cases} v_i - x & \text{if I win} \\ 0 & \text{otherwise} \end{cases}$$

(if I win depends on x.)

$G(x)$: Prob(second highest bid is at most x)

= Prob(win if I bid x)



$$F(x) = (v_i - x)\,G(x) + 0\,(1 - G(x))$$

"bid shading"

$$x^* = \operatorname{argmax} F(x)$$

2nd price auctions

$$f_2(x) = \begin{cases} v_i - (\text{second highest bid}) & \text{if I win.} \\ 0 & \text{if I lose} \end{cases}$$

Claim Dominant strategy for each bidder i is to bid their valuation

$$b_i^* = \operatorname*{argmax}_x F_2(x) = v_i$$

"strategy-proof"

Proof No matter what other people do,

bidding $b_i = v_i$ maximizes my reward.

Other people bid $\{b_1, \ldots\} = b_{-i}$

Let B denote $\max b_{-i}$

$b_i < B \Rightarrow$ utility is 0.

$b_i \geq B \Rightarrow$ utility is $v_i - B$

Case 1 pretend you bid $v_i < b_i$

you can do better by decreasing bid to $b_i = v_i$

if $b_i < B$ = 0 utility.

if $b_i \geq B$

Subcase $b_i > v_i \geq B \Rightarrow v_i - B \geq 0$

$b_i \geq B > v_i \Rightarrow v_i - B < 0$

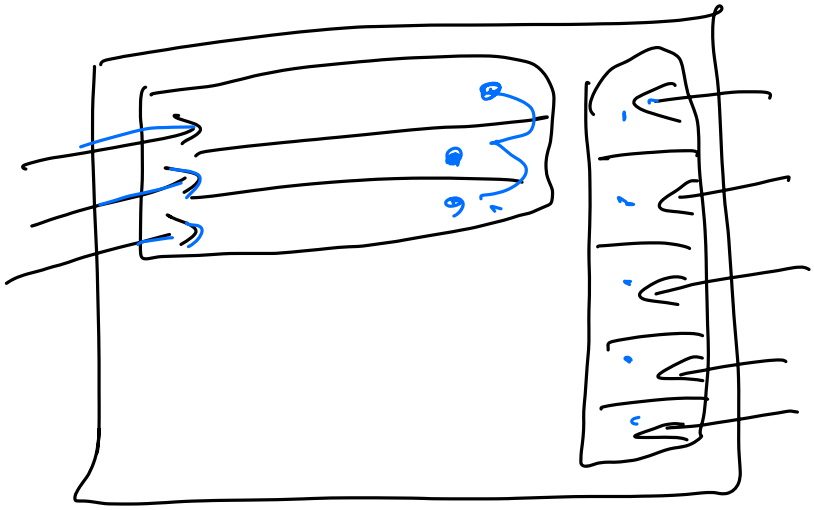
Case 2 pretend $v_i > b_i$

increasing my bid to v_i ↑ likelihood of winning

without changing what I pay.

Claim 2 "socially optimal" Person who values the item the most gets it if everyone bid truthfully.

Revenue equivalence principle.



"no

Generalized Second Price Auction

↳ Vickrey - Clarke - Groves

First Price Auctions

↳ "Credible"

2nd price auctions are not credible

Does there exist a strategy proof and credible auction?

No if you also want it to be static