

# Networks and Markets

Lecture 4: Centralized Matching Markets

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Susan Athey at a recent panel:

"online marketplaces have mostly been started by libertarians...the most consistent theme is that people think marketplaces will be fine just with prices, but then they realize they need to impose rules for the marketplace to succeed"

# Markets and Societal systems often fail

Why?

- Lack of *coordination* – selfish behavior by everyone makes everyone worse off. “Price of anarchy”
- Lack of *information/time* – individuals have constraints
- Fraud/other strategic behavior – manipulation by some makes others worse off
- Multiple equilibria, including some ‘bad’ ones

One role of the market designer/engineer is to prevent such failures

# An example: Market for lemons

George Akerlof, “The Market for Lemons: Quality Uncertainty and the Market Mechanism”

# Main Idea – single seller

- Suppose seller has a box  
With equal probability, either:
  - Nothing
  - Cup of coffee

Seller know what's in the box, but you don't

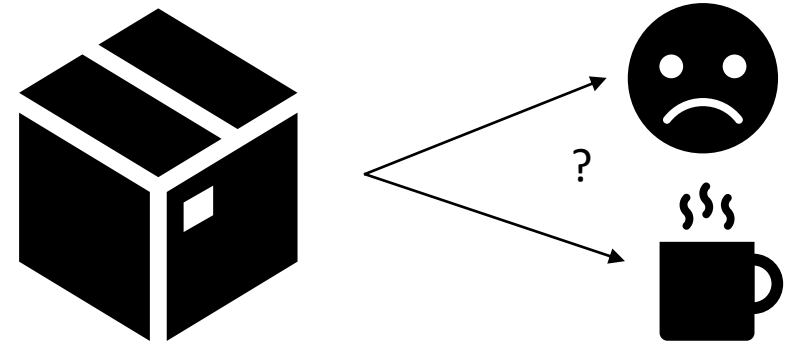
- Coffee is worth
  - \$1 to seller
  - \$1.50 to you

- At what price does a sale happen? <https://forms.gle/kqoVaVMzpDZ4q2LZA>

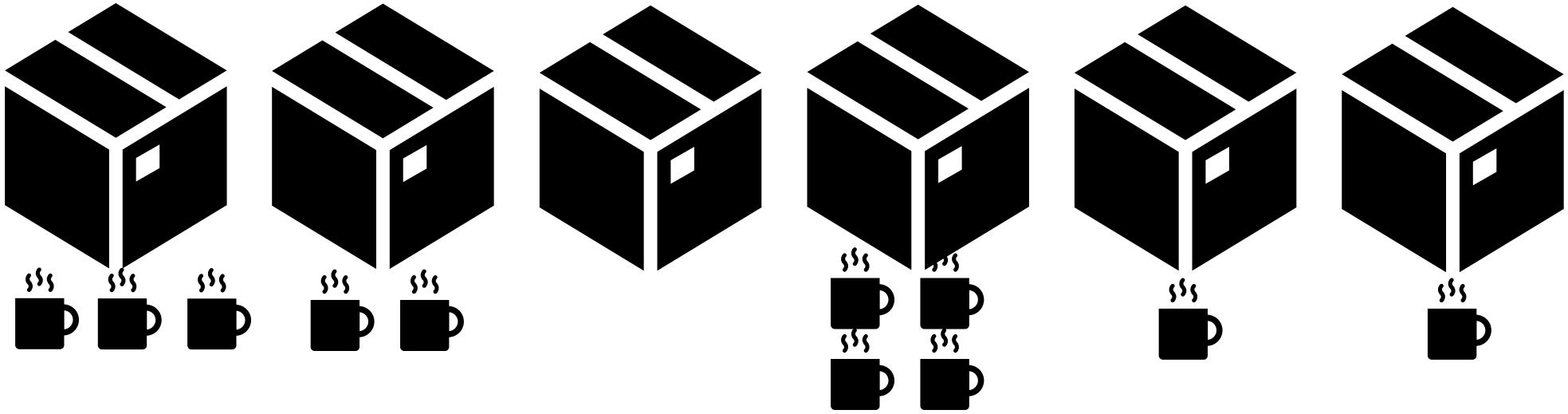
If I can guarantee coffee, you pay me somewhere in [\$1, \$1.50] and we're both happy

If I can't, you offer me \$0.75...Do I say yes?

Knowing the above, would you ever offer me \$0.75?



# Extending this to a market



Random between 0 and 4 cups  $\rightarrow$  you'd be willing to pay \$3 ( $2 \cdot 1.5$ )

$\rightarrow$  No one with 4 cups would accept

Between 0 and 3 cups  $\rightarrow$  you'd be willing to pay \$2.25 ( $1.5 \cdot 1.5$ )

$\rightarrow$  No one with 3 cups would accept

...Eventually only the empty box is left, the "lemon"

# Modern Example – Healthcare

Young & Healthy



Elderly and Sick



- What happens when insurers can't price discriminate based on status?

Average prices go up...so young & healthy flee the market

Ok...so you *mandate* that everyone buys insurance, and hope the mandate is strong enough

Of course...healthcare is a bad example of a free market. Mandate was effectively repealed, with little apparent effect.

# Main Ideas

## Main concepts:

1. Information Asymmetry & Adverse Selection
2. Price reveals information → rational expectations
3. Markets can *unravel*; converge to and then stuck in non-Pareto efficient allocations
4. How can design help us here? What if we had a reputation system with repeat transactions? Or an independent auditor?

## Markets for Lemons are everywhere

1. Healthcare
2. The sharing economy: eBay, AirBnB, Lyft, Upwork



More involved example:  
Residency matching

# Residency matching

In the United States (similar system in many countries):

- People do ~4 years of medical school
- Then, they do a *residency* in a specialty (pediatrics, internal medicine, surgery, etc).
  - (After residency, they may also do a *fellowship* for more specialized training)
- This is a very important system
  - Determines what kind of doctor you'll be!
  - Residents are also the core labor force of most hospitals/medical systems

# The market failure

Similar *unraveling* to what we saw above:

- Hospitals (especially less glamorous ones) started hiring future residents earlier and earlier during medical school (~1940s)
- In response, other hospitals also started hiring earlier. The market unraveling, such that students were being offered residency positions before they knew what they wanted to do (this is clearly bad)
- Many other labor markets have seen the same unraveling [Roth and Xing (1994)]
  - Law Clerk Market...

# Why is this bad?

- Less information -- offers have to happen early
- Exploding offers and not as "thick"
  - People have to accept offers without knowing other offers
- People play games of accept probabilities
  - Why should I interview someone who I am not going to get?  
=> then miss out on someone who would have come here.

# Initial solution desiderata

- In 1940s, a few attempts to fix the problem that didn't work well
- By 1951, a *centralized algorithm* would match doctors and hospitals

What are participants *action spaces*/what is the system *lever*?

- *Doctors* and *hospitals* each give ranked lists of the other side
  - Anyone can lie about their true preferences
- [Algorithm does the matching; assigns each doctor to a hospital]
- *Doctors* and *hospitals* can break their match
  - Suppose algorithm produces matches (A, X) and (B, Y). But Doctor A likes  $Y > X$ , and hospital Y likes  $A > B$ .
  - Then, (A,Y) can defect from the system and instead match with each other

# Initial solution: Gale-Shapley in 1 slide

- Desiderata:

- Doctors and Hospitals each submit *truthful* ranked lists
- After the algorithm produces its solution, there is no “blocking pair”: there exists no (A,Y) that would want to leave their assigned partner for each other

- How it works:



1: CBEAD

2 : ABECD

3 : DCBAE

4 : ACDBE

5 : ABDEC



A : 35214

B : 52143

C : 43512

D : 12345

E : 23415

[Gale Shapely on ipad]

# Summary of theory of Gale-Shapley

- Theorem: A stable matching always exists [No “blocking pairs”]
- Theorem: Gale-Shapley algorithm finds a stable matching
- Theorem: Participants don’t want to lie about their preferences, *but only the “proposing” side.*

Problem solved? By 1970s, people started complaining again...

- Increase in *couples* in medical schools wanting to match together
- Implementation of algorithm favors hospitals? (Over doctors?)
- Concerns about strategic behavior by doctors



# The redesign of the market (1990s)

Favoring doctors over favoring hospitals:

“Simple” change to algorithm: have doctors “propose” instead of hospitals. Rest of theoretical results hold. (And now, it’s strategy-proof for doctors!).

Couples: harder case -- Stable matching may not exist with couples

BUT, some hope:

- Computational simulations suggest that in practice stable matchings almost always exist
- Follow up theory (e.g., Nguyen & Vohra 2018): “for any student preferences, we show that each instance of a matching problem has a “nearby” instance with a stable matching”

# Problem solved forever?

Theory differs from practice in many ways

- Empirically, many doctors still mis-report their preferences. Why?
- The algorithm assumes people *know everything* about the other side, and can rank as many as they want
  - In practice, this is not true: you must *interview* at a school to even have a chance. So, if you only interview at top schools and don't get any...
  - Opposite problem: what if the same top 10 doctors interview everywhere (50 hospitals, each with 1 position)?
    - Evidence that this happened last few years due to Zoom interviewing
    - Recent proposal to have a centralized matching system for interviews: [Explaining a Potential Interview Match for Graduate Medical Education | Journal of Graduate Medical Education \(allenpress.com\)](#)

# Lessons from residency matching

- Systems can fail due to seemingly minor incentives issues (causes bad equilibria)
- Designing even a limited system is hard and often politically tense
- Interplay between theory, empirical analysis, and simulation
  - “It turned out that the simple theory offered a surprisingly good guide to the design, and approximated the properties of the large, complex markets fairly well. Field and laboratory data showed that the static idea of stability went a long way towards predicting which kinds of clearinghouse could halt the dynamics of unraveling. And computation showed that many of the departures from the simple theory were small, and that some of the most severe problems that the counterexamples anticipated, such as the possibility that no stable matching would exist, were rare in large markets. Computation also revealed that large markets could achieve even nicer incentive properties than anticipated by the simple theory.”
- Objective functions matter! (Doctor vs hospital optimal)
- People often don't behave “optimally,” but that doesn't make theory useless
- Even if one component of the system is provably optimal, surrounding components (here, interview process) might undo benefits

# Some nice quotes from Al Roth (2002)

- The largest lesson in all this is that design is important because markets don't always grow like weeds—some of them are hothouse orchids. Time and place have to be established, related goods need to be assembled, or related markets linked so that complementarities can be handled, incentive problems have to be overcome, etc. If game theory is going to be as important a part of design economics as it is a part of economic theory, we'll have to develop tools not just to develop conceptual insights from simple models, but also to understand how to deal with these complications of real markets.
- In the long term, the real test of our success will be not merely how well we understand the general principles that govern economic interactions, but how well we can bring this knowledge to bear on practical questions of microeconomic engineering. Just as chemical engineers are called upon not merely to understand the principles that govern chemical plants, but to design them, and just as physicians aim not merely to understand the biological causes of disease, but their treatment and prevention, a measure of the success of microeconomics will be the extent to which it becomes the source of practical advice, solidly grounded in well tested theory, on designing the institutions through which we interact with one another.

# So what has changed since 2002?

- Rise of online marketplaces (and more generally, computational systems) has made the type of work he described ubiquitous
- What's difference? These applications are far “messier”
  - Faster (many times a second, as opposed to once a year)
  - Many complexities; not a ‘closed’ system. (in ride-hailing, pricing affects matching affects wages affects long term driver capacity...)
- The field is more mature, new methods, incorporation of data science
- Critiques of approach, as enters more sensitive areas